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Review of Space Debris Modeling Methods and Development Direction of the Korean Space Debris Models

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Space debris poses significant threats to spacecraft and human activities in space. Accurate modeling of space debris is crucial for understanding and mitigating these risks, ensuring the sustainability of the space environment. This paper discusses the importance of space debris modeling in the space environment, highlighting its critical role in safeguarding assets in orbit. Two primary methods of space debris modeling, namely the 1D and 3D approaches, are discussed in detail, and their respective strengths and limitations are elucidated. Furthermore, a comprehensive review of existing models, including the space debris evolutionary model (MOCAT, SOLEM, DAMAGE, LEODEEM & GEODEEM, DELTA, and LEGEND) and engineering models (MOCAT-MC, NEODEEM, MASTER, ORDEM), are presented. These models offer valuable insights into the dynamics and characteristics of space debris populations, aiding in formulating effective debris mitigation strategies and orbital capacity problems for reducing the possibilities of Kessler's syndrome. Additionally, the paper provides insights into the ongoing development of the Korean space debris model, focusing on its methodology and space debris cataloging techniques for modeling space debris environments.

Keywords: space debris, space debris environment, evolutionary model, engineering model, Korean space debris model

1. INTRODUCTION

Space debris modeling has become critical research in the past 20 years as the number of satellites, specifically in the low Earth orbit (LEO), has increased uncontrollably. Additionally, with several private companies and government-centric space agencies announcing plans to develop/operate satellite constellations within different orbital planes and altitudes, modeling the space environments to understand the effect of placing such a large number of satellites has become important research.

Several methods exist to assess the risks posed by space debris. The quantification of these risks can also be viewed in multiple ways, starting from single-to-single spacecraft, multiple-to-single spacecraft, and lastly, multiple-to-multiple spacecraft, denoting microscopic to macroscopic view of quantifying the space debris risks to the environment (Kim et al. 2022). The single-to-single spacecraft collision method utilizes the propagated uncertainties (covariance) to calculate the collision probability at the time-of-closest approach (TCA; Patera 2001, 2008). The multiple-to-single spacecraft method calculates the spacecraft's collision probability within a given period using the debris flux information, typically pre-determined based on the cataloged spacecraft in a 3D spherical cell (Klinkrad 2006). The multiple-to-multiple spacecraft collision method is typically used to determine the interaction of two distinct satellite constellations. This article focuses on the macroscopic view, which considers multiple-to-single and multiple-to-multiple collisions.

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Space debris models with a macroscopic view can be categorized into two distinct types: *evolutionary* and *engineering* models. The evolutionary model describes the evolution of space debris in the space environment by adding several artificial factors (newly launched spacecraft, collision, fragmentation) and natural factors (solar radiation pressure, atmospheric drag).

The evolutionary model has been studied by many space agencies and universities worldwide. Astrodynamics, Space Robotics, and Controls Laboratory (ARCLab) of the Massachusetts Institute of Technology (MIT) developed the MIT orbital capacity assessment tool (MOCAT) to assess the risk-based capacity as a function of altitude. The MOCAT program utilizes the source-sink method, which solves a differential equation of newly launched, derelict, and deorbiting spacecraft to determine the orbital capacity as a function of altitude (D'Ambrosio et al. 2022). Astronautics Research Group at the University of Southampton has developed a space debris evolutionary model named DAMAGE (Lewis et al. 2001, 2011). The DAMAGE analysis tool aims to account for the unique characteristics to determine the collision probability and evolution of the space debris within the Geosynchronous region. Space object long-term evolution model (SOLEM; Wang & Liu 2019) is China's first comprehensive debris long-term evolution model. SOLEM adopts a source-sink model similar to that used in MOCAT. Near-Earth orbital debris environment evolutionary model (NEODEEM; Ariyoshi & Hanada 2009) was developed by Kyushu University and Japan Aerospace Exploration Agency. Moreover, debris environment long-term analysis (DELTA; Martin et al. 2004) and NASA LEO-to-geostationary Earth orbit (GEO) environment debris (LEGEND; Krisko 2003) from the European Space Agency (ESA) and the National Aeronautics and Space Administration (NASA) have made significant improvements in the space debris evolutionary modeling methods. All the models listed above tried to incorporate various types of perturbation methods to determine the evolution of space debris more effectively and accurately.

Using the debris evolution models, ESA and NASA have developed the engineering model, which takes in the user's spacecraft orbital information and outputs the yearly flux information, which is crucial for calculating the collision probability. By utilizing their own debris evolutionary models, ESA has developed meteoroid and space debris terrestrial environment reference (MASTER; Klinkrad et al. 1997), and NASA has developed the orbital debris engineering model (ORDEM; Krisko 2014). Alongside the currently developing space debris evolutionary and engineering models, Korea Astronomy and Space Science Institute (KASI) and Korea Advanced Institute of Science and Technology (KAIST) are developing Korea's first space debris engineering model, which aims to utilize the 3D cell method to provide yearly flux information to the user with predefined spacecraft orbit.

Section 2 discusses two common space debris modeling methods in detail: the 1D source-sink model and the 3D cell method. Section 3 compares various space debris evolutionary models developed by different space agencies and universities. Section 4 discusses the currently developed space debris engineering models that work alongside the space debris evolutionary models discussed in Section 3. Finally, Section 5 discusses the Korean space debris modeling method and describes its significance compared to currently developed space debris evolutionary and engineering models.

1.1 Space Debris Evolutionary Models

Space debris evolutionary models aim to propagate the known space objects and newly categorized information to understand the impact of the space debris at the region of interest. Fig. 1 shows the list of space debris evolutionary models developed in various agencies and universities, where the x-axis defines the dimension of the space debris environment, and the y-axis defines the altitude of the space debris evolution. A total of 6 space debris evolutionary models are discussed and explained in detail in subsequent subchapters.

1.1.1 MIT Orbital Capacity Assessment Tool

The MIT Orbital Capacity Assessment Tool, abbreviated as MOCAT (D'Ambrosio et al. 2022), is the probabilistic source-sink model developed as a part of MIT's orbital debris models. MOCAT aims to model the LEO region's space debris orbital capacity utilizing active satellites,

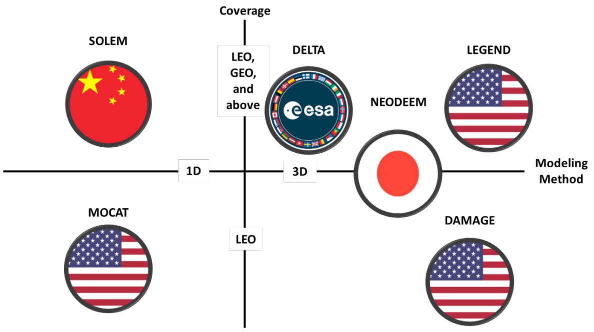

Fig. 1. Space debris evolutionary model with x-axis for modeling method, and y-axis for the altitude coverage.

derelict satellites, and categorized debris. The orbital capacity refers to the maximum allowable number of spacecraft within a specified altitude shell. The shells are divided with an equal altitude, creating an empty sphere-like shape with an altitude range for describing the number of spacecraft within each shell. The shell method utilized by MOCAT aims to provide a sense of the current congestion level at the LEO region, aiding decisions such as launching new spacecraft into the specific shell (altitude) and providing strategic launch scheduling for future missions. Orbital capacity is typically categorized into two main types: intrinsic capacity and risk-based capacity (Jang et al. 2023). Intrinsic capacity refers to the capability of placing active space objects (ASOs) in a specific area of space in a compatible manner to avoid collisions indefinitely. This definition primarily focuses on space traffic management and does not account for debris or non-compliant ASOs. On the other hand, risk-based capacity considers factors like collisions and other unpredictable events over time. There is no universally accepted metric for measuring risk-based capacity, but various definitions have been proposed. For instance, the environmental consequences of orbital breakups (ECOB) index assesses the consequences and likelihood of orbital fragmentation. Other definitions, like the number-time (NT) product, analyze the trend in fragment numbers, while indexes such as the criticality of the spacecraft index (CSI) evaluate the environmental impact of larger space objects by considering parameters like mass, lifetime, spatial density, and inclination.

1.1.2 Space Object Long-Term Evolution Model

SOLEM developed by the National Astronomical Observatory and the Space Debris Observation and Data Applications Center of China is a LEO space debris long-term evolutionary model utilizing a 1D source-sink model and is currently under development together with China National Space Administration. SOLEM is a predictive model for space debris that forecasts population trends, estimates collision rates among space objects, and analyzes the impacts of mitigation measures and uncertainties on debris evolution. The SOLEM model utilizes a simplified semi-analytical orbital propagator, where integration is performed on perturbation functions with short-periodic terms excluded. In near-circular orbits, the primary perturbations consist of Earth's nonspherical gravity perturbation (e.g., J2, J3, J4) and atmospheric drag. In high eccentricity orbits, in addition to Earth's nonspherical gravity and atmospheric drag, perturbations arising from solar radiation pressure and the gravitational influence of the Sun and Moon are also accounted for. A small space debris population was used to validate the orbital propagator used in SOLEM. This experiment aimed to compare the results of SOLEM's orbital propagation with historical data regarding the evolution of a small population from a statistical perspective. The experiment utilized all 1,021 cataloged LEO-crossing objects as of January 1, 1980. SOLEM utilizes NASA's standard breakup model to simulate the creation of fragments resulting from in-orbit breakups. This model, endorsed by NASA, is widely recognized and accepted as the predominant method for generating fragments in current simulations of space debris, which can be found in this paper (Johnson et al. 2001; Krisko 2011). SOLEM's current orbital scope spans 200 km to 2,000 km in the LEO region. However, they plan to extend the SOLEM's coverage beyond LEO to include the GEO region. Additionally, there will be improvements to the post-mission disposal (PMD) model, refining both the selection process for disposed orbits and the computational efficiency.

1.1.3 DAMAGE

The DAMAGE software analysis tool developed at the University of Southampton's Astronautics Research Group aims to model debris in High Earth orbits over a long period. The goal of the DAMAGE project is to incorporate the distinctive features relevant to modeling the GEO environment. Utilizing the model, various applications, such as exploring mitigation strategies for the GEO region, were tested and compared to that of other space debris evolutionary models.

DAMAGE can model space debris environments between 2,000 km to super-GEO (GEO + 2,000 km); hence, the tool does not only include space debris but also meteoroids at high altitudes near the GEO belt. The new DAMAGE model includes collision risk assessment tools that can be applied individually or together, depending on the specific use case. Its main applications include assessing risks to spacecraft in high Earth orbit (HEO), analyzing the long-term stability of the HEO environment, evaluating various mitigation methods, and assessing proposed and novel spacecraft disposal strategies. Unlike MOCAT and SOLEM, which utilize spherical shells as a 1D debris model, DAMAGE utilizes spherical cell methods, dividing the space into a small cell of a sphere with varying altitude, azimuth, and elevation angle (Lewis et al. 2011). DAMAGE accounts for the long-term evolution of space debris in such high altitudes by including the spherical harmonics of the Earth and the solar radiation pressure, which create oscillatory motion in the eccentricity of the spacecraft. Lastly, there is Luni-solar gravitational attraction, as the Moon's gravity becomes more significant than that of the spherical harmonics of the Earth at the GEO belt (Lewis et al. 2001). DAMAGE software also models the breakup events, such as the explosion/collision of spacecraft. It calculates the number of debris generated based on the mass, size, and relative velocities of the two spacecraft. Unlike the LEO region, where the difference in velocity is near 10 km/s, satellites in the GEO belt have a difference in velocity of less than 3 km/s. DAMAGE models the spacecraft collision between GEO satellites to be a non-catastrophic event, whereas the collision between GEO and geosynchronous transfer orbit (GTO) spacecraft is catastrophic. DAMAGE software was validated by evolving the space debris environment from 1957 to the reference epoch, and comparing the modeled data with the historical data and results using MASTER/IDES.

1.1.4 Near-Earth Orbital Debris Environment Evolutionary Model

Kyushu University and Japan Aerospace Exploration Agency (JAXA) have collaborated to create NEODEEM (Ariyoshi & Hanada 2009; Harada et al. 2023), a predictive model for assessing environmental changes in near-Earth orbit caused by debris. This model accounts for the dynamics of objects larger than 10 cm, incorporating factors like geopotential, gravitational forces from celestial bodies like the Sun and Moon, solar radiation pressure, and atmospheric drag to project orbital trajectories. Similar to the DAMAGE software, NEODEEM adapts NASA's standard breakup model. If the ratio of the smaller object's relative kinetic energy to the larger object's mass is 40 J/g or greater, the collision is deemed catastrophic. NEODEEM annually assesses explosion events, with an explosion being presumed to happen if a randomly generated number falls below each spacecraft's explosion probability. Fragments exceeding 10 cm are produced based on the standard NASA breakup model. The likelihood of an explosion is defined as 0.001. To ensure accuracy, NEODEEM conducts an average of 100 Monte Carlo simulations due to its utilization of a random number generator. The initial population for the simulation consists of a database file compiled by JAXA and contains information on all cataloged orbital objects larger than 10 cm as of January 1, 2021. This file incorporates various details about each object, such as its orbit, mass, and surface area. The data was gathered from sources including two line elements (TLE) obtained from Space Track, observations made with JAXA telescopes, and breakup models. As the TLE data lacks mass and area information for individual objects, this data for intact objects is supplemented through literature research. Mass characteristics for fragmented objects are randomly assigned to correspond with the estimated area-to-mass ratio (A/M) based on fragments generated by the NASA standard breakup model.

1.1.5 Debris Environment Long-Term Analysis

The European Space Agency's DELTA tool is utilized to study the extended propagation and evolution of space debris from Very LEO to orbital altitude above the Moon (400,000 km). DELTA contributes to inter-agency space debris coordination committee (IADC) investigations on long-term evolution, which have shaped mitigation guidelines and emphasized the necessity of active debris removal (ADR). DELTA, a three-dimensional semi-deterministic model, enables users to explore space debris evolution and associated collision risks across low, medium, and geosynchronous Earth orbits over custom timeframes. It assesses the enduring impacts of diverse future traffic patterns and debris mitigation strategies, including passivation and end-of-life disposal, while also considering remediation efforts such as ADR across various scenarios and criteria. DELTA utilizes an initial population of space objects to predict the evolution of those larger than a specified size set by the user. These objects, described by representative samples, are propagated using a fast analytical orbit propagator that considers major perturbations. Typically, the initial population is derived from ESA's MASTER-2009 model and can include objects as small as 1 mm. DELTA incorporates detailed future traffic models for various activities (e.g., launches, explosions, and solid rocket motor firings) based on historical data. Collision predictions employ a target-centered approach to stochastically forecast impacts among all objects within the DELTA population. Fragmentation events are modeled using the EVOLVE 4.0 model from NASA. This paper provides a detailed overview of DELTA's architecture and explains its unique flux-based method for computing collision probabilities, differing from the commonly used CUBE method in other long-term evolution tools. DELTA software allows the users to perform analysis such as the influence of solar activity on long-term evolution (Radtke & Stoll 2016), the effect of ADR for LEO missions (Virgili & Krag 2013), and even the level of compliance for mega-constellation (Virgili & Krag 2015) for optimal constellation deployments (Sung & Ahn 2023). DELTA undergoes continuous improvement and revision, with a current focus on developing alternative collision detection algorithms and enhancing the utilization of atmospheric models while maintaining performance standards.

1.1.6 The NASA LEO-to-GEO Environment Debris

LEGEND (Liou et al. 2004), known as the LEO-to-GEO Environment Debris model, is a comprehensive three-dimensional model for the evolution of space debris developed by NASA to replace the older version of space debris evolutionary mode, EVOLVE. It spans altitudes from 200 to 40,000 km, encompassing regions such as LEO, medium Earth orbit (MEO), GEO, and beyond. The model provides detailed characteristics of debris, including size distribution, spatial density, velocity distribution, flux over time, altitude, longitude, and latitude. LEGEND comprises both historical simulations from 1957 to 2001 and future projections. The primary purpose of the LEGEND historical component is to replicate the debris environment from 1957 to 2001. This involves utilizing an updated satellite launch database (DBS-database), along with two efficient propagators (PROP3D, GEOPROP) and a new NASA satellite breakup model. The program structure, input, and output options are optimized for reasonable execution time and manageable file storage space. It offers multi-dimensional descriptions of the debris environment and includes additional analysis modules for visualizing outputs and facilitating comparisons with ground-based debris observations. The primary program is coded in FORTRAN90, with supplementary analysis tools developed in interactive data language. LEGEND generates debris distributions in various formats, including 1D (altitude), 2D (altitude, latitude), and 3D (altitude, latitude, longitude) at specific times or over time intervals. The program processes orbital element arrays through multiple modules to analyze debris properties across multiple dimensions. Typically, the 2D distributions based on altitude and latitude are adequate for defining and characterizing the debris environment. However, exceptions arise with GEO objects or recent fragments from breakup events in near Sun-synchronous orbits, where including longitude dependence may be necessary for accurately describing their spatial distribution.

1.2 Space Debris Engineering Model

The space debris engineering model utilizes the evolutionary models discussed in the prior chapter. Up to date, only two space debris engineering models have been developed, which are MASTER developed by ESA with DELTA as their fundamental space debris evolutionary model. And ORDEM (developed by NASA) utilized LEGEND as their fundamental space debris evolutionary model (Fig. 2). Two space debris engineering models are discussed in greater detail in the following subchapter.

1.2.1 Meteoroid and Space Debris Terrestrial Environment Reference

The Meteoroid and Space Debris Terrestrial Environment Reference, abbreviated as MASTER, is a space debris engineering model developed by ESA. The European MASTER model originated in 1987 after the breakup of an Ariane upper stage in 1986. This event prompted the need to comprehend its cause, leading to the development of MASTER. The model's core principle involves simulating all known space debris-generating events to establish a synthetic population, aiding in assessing space debris flux for satellite missions and facilitating risk evaluations. Additionally, MASTER enables predictions of the environment's evolution by accurately describing its dynamics. Over the last two decades, major space agencies have utilized MASTER to assess mitigation measures' effectiveness and advocate for necessary actions. The initial beta version of MASTER was released in 1995 and was made accessible to a select group of experts. This version focused solely on the LEO region and accounted for space debris larger than 0.1 mm resulting from launch activity, explosions, and collisions from 106 previous breakup incidents. Additionally, to assess the risk of impacts from the natural meteoroid environment, the Grün model was incorporated. MASTER is currently utilized in mission planning and risk evaluation endeavors. It serves as either a standalone tool or as a resource to provide foundational data for more specialized applications. For instance, the debris risk assessment and mitigation analysis (DRAMA) suite leverages MASTER through its assessment of risk event

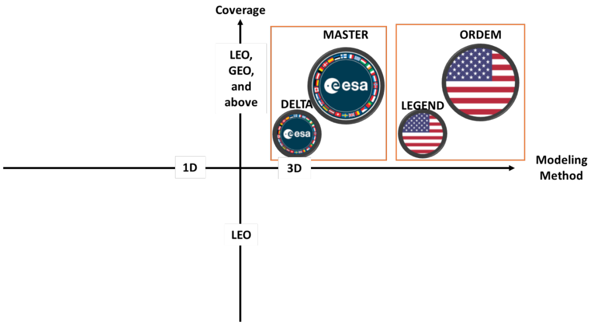

Fig. 2. Space debris engineering model with their respective evolutionary mode. Small circles define the space debris evolutionary models utilized for the large circle, which defines the engineering models.

statistics (ARES) and MASTER-based impact and damage assessment software (MIDAS) components. Additionally, MASTER supports vulnerability analyses in various platforms such as Systema-Debris, ESABASE2/Debris, or the particle impact risk and vulnerability analysis tool (PIRAT). Moreover, beyond its role in space missions and engineering, MASTER finds extensive usage in academic research by insurers, economists, and other entities seeking insights into the space debris environment. The MASTER software also provides asteroid flux information within the super GEO regions. According to (Sdunnus et al. 2001), MASTER software is expected to include the flux information for the cis-lunar region (Russell & Angelopoulos 2014), supporting future lunar missions or trajectories that utilize third-body perturbation (Lee & Ahn 2021, 2024b; Lee et al. 2023) along with electrically propelled missions (Lee & Ahn 2023a, 2023b, 2024a) with slow orbital altitude increase.

The MASTER population serves as an event-driven simulation encompassing all known events that generate debris, including objects listed in the U.S. Space Surveillance Network (SSN) catalog, covering items with diameters down to around 10 cm in LEO and 1 m in GEO. Various models are employed to simulate artificial objects and their orbital trajectories over time, termed "sources," which attribute an origin to each object. These sources comprise fragments, solid rocket motor (SRM) slag and dust, sodium-potassium (NaK) droplets, paint flakes, ejecta, and multi-layer insulation (MLI) objects. Objects from each source are characterized by distinct release mechanisms, orbital distributions, material compositions, and size and mass distributions. Calibration of the model involves utilizing dedicated radar and telescope observation data for objects larger than 1 cm in LEO and above 10 cm in GEO. For smaller objects below 1 cm, impact data from returned surfaces are analyzed. Due to the prevalence of fragments in the > 1 cm population, the fragmentation event database has been updated to include new events and re-evaluate past incidents, notably focusing on the Fengyun-1C (FY-1C) anti-satellite test from 2007 and the Cosmos-Iridium collision event from 2009 due to their significant impact on the fragment population. As of November 1, 2016, the two largest fragmentations, in terms of tracked debris count, are the Briz-M explosion in 2012 and the National Oceanic and Atmospheric Administration (NOAA) 16 explosion in 2015, with a total of 261 confirmed fragmentations in the database up to that date.

The MASTER program offers a unique benefit whereby users can incorporate additional satellite constellations into existing space debris models to assess the potential rise in collision risk for individual satellites or groups of satellites. For instance, users can analyze the yearly increase in three-dimensional debris density resulting from the continual deployment of StarLink satellites. Furthermore, the software enables users to visualize spatial data in both three-dimensional and two-dimensional formats using its integrated plotting tools.

1.2.2 The Orbital Debris Engineering Model

ORDEM developed by NASA starting mid-1990 has been the standing standard for space debris modeling methods verification and validation. ORDEM's primary function is to provide fluxes of debris per square meter per year for a given time. These fluxes are determined based on cumulative size, meaning they are reported for a specific size and larger. This approach is rooted in risk assessment principles, where the impact of a debris particle of a certain size capable of critically damaging a spacecraft component implies that larger debris would also pose a threat. ORDEM calculates and presents cumulative fluxes using eleven half-decade size thresholds, or fiducial points, ranging from 10 μm to 1 m. Fluxes at sizes between these thresholds are derived through interpolation. Debris fluxes are modeled for objects larger than 10 μm in LEO, specifically at altitudes below 2,000 km, and for objects larger than 10 cm in GEO. It's important to note that while GTO and GEO orbits may physically overlap, the dynamics, including perturbation forces and impact velocities, as well as the physical size and structure of satellites within the GEO region, are distinct. Therefore, ORDEM only provides debris fluxes in GEO for sizes of 10 cm and above. Fluxes below 10 cm at altitudes above LEO are solely attributed to high-eccentricity debris sources.

ORDEM operates as a data-centric model, leveraging measurement data from ground-based radar and optical sources as well as in situ sources to calibrate initial reference models of the orbital debris environment. The LEGEND model serves as the foundation for most sub-populations within ORDEM, encompassing numerous orbits with defined orbital elements and characteristics such as size, mass, area-to-mass ratio, and material type for each object. The historical population is derived from a database maintained by the NASA drbital debris program office (ODPO), capturing information on launches, breakups, and maneuvers. Fragments from confirmed historical fragmentation events are generated using a specialized version of the NASA satellite structure breakup model (SSBM), incorporating material density assignments for fragments smaller than 10 cm based on analyses of SOCIT series fragments and known satellite material breakdowns. LEGEND models fragments down to 1 mm in diameter in

LEO and assigns material densities accordingly, while in GEO, fragments down to 10 cm are modeled with material density designations. For future projections in ORDEM 3.1 (spanning from 2016 to 2050), objects were added to the population, assuming a repetition of the previous 8-year launch traffic cycle and a PMD success rate of 90% for rocket bodies and spacecraft. Future collisions and explosions were statistically modeled, with collisions between objects larger than 10 cm assessed according to the "cube" collision assessment algorithm in LEGEND.

2. SPACE DEBRIS MODELING METHODS

2.1 1D Source-Sink Method

The 1D source-sink method for modeling space debris utilizes a set of coupled ordinary differential equations to describe the space debris evolution model. The model is considered a 1D model as the space environment is subdivided into a spherical shell with a predefined altitude (Fig. 3).

The source-sink model utilizes the following ordinary differential equation to propagate the whole population ($\mathbf{P}(t,h)$ = [$S(t,h)$, $D(t,h)$, $N(t,h)$]) at a specific time (t) and altitude (h) as follows:

$$\dot{\mathbf{P}} = \dot{\mathbf{\Lambda}} + \dot{\mathbf{C}}_{PMD} + \dot{\mathbf{C}} + \dot{\mathbf{F}} \tag{1}$$

In Eq. (1), $\dot{\mathbf{P}}$ defines the time derivative of the whole population. The time and altitude are omitted for simplicity onward; however, note that all of the functions provided below are functions of time and altitude. The population function contains active satellites (S), derelict satellites (D), and debris (N) at each altitude at a given time. Eq. (1) shows four independent terms used to propagate the space debris population size forward probabilistically.

$\dot{\mathbf{\Lambda}}$ represents new launches as a function of time, which is formulated as follows

$$\dot{\mathbf{\Lambda}} = [\lambda, 0, 0] \tag{2}$$

The new launch time derivative only accounts for the active satellites, assuming the new launch has no probability of collision during launch.

$\dot{\mathbf{C}}_{PMD}$ is the time derivative of the spacecraft with the PMD capability within the system. The spacecraft with PMD capabilities is considered only during the its operational period; it is removed (deorbited to the atmosphere) from the active satellites if successful or considered a derelict satellite if unsuccessful. Hence, the effect of the PMD on the space debris population is modeled as follows:

$$\dot{\mathbf{C}}_{PMD} = \left[-\frac{S}{\Delta t}, \frac{(1-P_M)S}{\Delta t}, 0 \right] \tag{3}$$

In this equation, S is the active satellite population information from the state, and P_m defines the PMD's success probability. Note that with the PMD's success, the active satellite's population decreases (hence the negative sign) but increases as the PMD fails as a function of the active satellite. PMD is assumed to have no impact on the population of the debris; therefore, dedicated PMD missions are not considered in this model.

$\dot{\mathbf{C}}$ defines the collision between the populated spacecraft

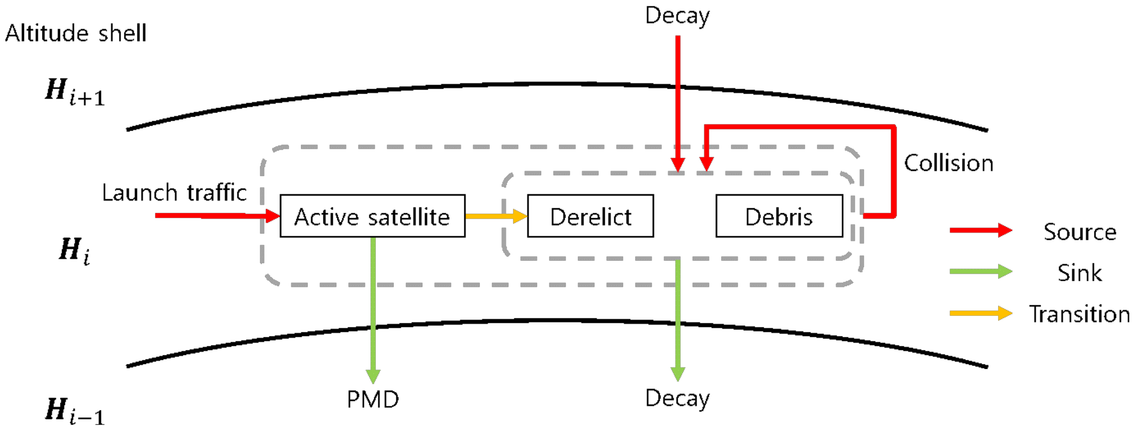

Fig. 3. 1D Source-sink space debris modeling method. Adapted from D'Ambrosio et al. (2022) with CC-BY 4.0.

modeled as follows

$$\dot{\mathbf{C}} = \left[\dot{C}_S, \dot{C}_D, \dot{C}_N \cdot n_f \right] \tag{4}$$

The collision between different populations is modeled by using the kinetic theory of gases as follows.

$$\dot{C}_i = \sum_{j=1}^{N} \Gamma_{ij} \phi_{ij} C_i C_j \tag{5}$$

where i and j are the subscripts for the two space debris. Γ_{ij} defines the collision avoidance probability in between the state variables and ϕ_{ij} defines the collision probability of a specific altitude within a given time. The collision probability for each species is defined as

$$\phi_{ij} = \pi \frac{v_r(h) \sigma_{ij}}{V(h)} \tag{6}$$

In this equation, $v_r(h)$ and $V(h)$ define the average relative velocities and the volume of the shell as a function of altitude. σ_{ij} is the impact parameter affected by altitude, time, and debris category. In this study, the parameter is defined as a function of the radius of the object in each spherical shell as follows.

$$\sigma_{ij} = (r_i + r_j)^2 \tag{7}$$

Γ defines the level of effect the collision in between species can cause to the debris environment, defined as follows

$$\Gamma = \begin{bmatrix} -\alpha_a & -(\delta+\alpha) & -(\delta+\alpha) \\ \delta & -1 & -1 \\ \alpha & \alpha & 1 \end{bmatrix} \tag{8}$$

where α_a and α defines the fractions of collisions for active satellites and δ defines the ratio of the density of disabling to lethal debris. The sign of each element in Γ corresponds to the increase or decrease in the debris species for corresponding collisions between species.

Lastly, n_f in Eq. (4) is the number of fragments generated based on the severity of the collision between species. The source-sink model utilizes the NASA standard breakup model (Johnson et al. 2001). The number of fragments generated during the collisions is determined based on whether the collision is catastrophic (f,c) or non-catastrophic (f,nc). The number of fragments generated is formulated as follows

$$\begin{aligned} n_{f,c} &= 0.1 L_C^{-1.71} \left(M_i + M_j \right)^{0.75} \\ n_{f,nc} &= 0.1 L_C^{-1.71} \left(M_i \cdot v_{imp}^2 \right)^{0.75} \end{aligned} \tag{9}$$

where L_c defines the characteristic length of the space debris, M defines the masses of the species, and v_{imp} defines the impact velocity, which is assumed to be an average circular orbit velocity as a function of altitude.

The last term in Eq. (1) defines the drag effect on the space debris, which decreases the orbital altitude as a function of time for derelict and debris populations. We assume that the active satellites continuously perform station-keeping maneuvers to compensate for the drag effect, orbiting at an identical altitude throughout the spacecraft's lifespan. The drag effects are formulated as follows.

$$\dot{\mathbf{F}} = \left[0, \dot{F}_{d,D}, \dot{F}_{d,N} \right] \tag{10}$$

An in-depth analysis of the source-sink model can be found in (Lewis et al. 2011; Wang & Liu 2019; D'Ambrosio et al. 2022).

2.2 3D Cell Method

The 3D cell method for space debris modeling utilizes propagated state information of the orbiting body and calculates the flux of the 3D cells of the modelled sphere. The 3D cell method calculates the collision probability based on the laws of kinetic gas theory and is calculated as follows

$$c = v D A_c \Delta \mathrm{t} \tag{11}$$

where c defines the mean number of collisions encountered by an object within the same cell with a cross-section area of A_c, moving through a cell with a uniform particle density of D, during the spacecraft propagated duration of Δt with a constant velocity, v. In Eq. (11), the impact flux of the spacecraft within Δt is F=vD in the unit of $\mathrm{m}^{-2}\mathrm{s}^{-1}$.

The probability of experiencing n number of collisions throughout the propagated time is calculated utilizing the mean number of collisions as follows.

$$P_{i=n} = \frac{c^n}{n!} \exp(-c) \tag{12}$$

Hence, the probability of experiencing no collision throughout Δt is calculated as

$$P_{i=0} = \exp(-c) \tag{13}$$

The probability of one or more collisions throughout the same Δt is calculated as follows.

$$P_{i\geq 1} = 1 - \exp\left(-c\right) \simeq c \tag{14}$$

The 3D cell method's spherical cells are divided by a spherical volume cell centered at the origin of the Earth-inertial-frame (ECI). The volume cell is divided by geocentric distance, r_i, declination measured from the equatorial plane, δ_j, and planar angle measured from the vernal equinox as α_k, where the subscript *i, j*, and *k* denotes the level of sensitivity. More coarse indexing results in higher spatial resolution of the flux information, increasing the flux accuracy with the cost of computational complexity. The 3D cell method is utilized by NASA's ORDEM and ESA's MASTER program and their respective space debris evolutionary models. Both software utilize approximated mathematical models to propagate non-trackable and trackable space debris, which is discussed next.

2.2.1 Trackable Space Debris

The 3D cell method utilizes the cataloged database from TLE information and ground observation data to generate flux information with trackable space debris. According to (Krisko et al. 2015), the detection size thresholds of operational space surveillance systems are predominantly between 1 cm in LEO and 0.7 cm in GEO, where the objects are either intact spacecraft, upper stages of launch vehicles, or other types of known and unknown space debris. The detection capabilities of the US SSN catalog are limited, particularly for objects with diameters of 10 cm or larger. It's anticipated that the actual number of objects in this size range is significantly higher, potentially by a factor of three, due mainly to undetected fragments. This estimation finds support from optical observations in LEO and radar observations down to 5 cm diameters, facilitated by integrating the Cobra Dane phased array radar into the SSN in 2002. While the lower size range of the updated catalog remains incomplete, there's an expected improvement in our understanding of the population with diameters of 10 cm or larger. The reported number of objects in the extended SSN catalog, around 14,000, aligns with trends observed independently through optical and experimental radar observations. However, among the two primary populations of trackable space objects—intact objects and debris—fragmentation debris is inadequately monitored from ground-based observations. The tracking of space objects can be divided into two main categories: intact objects and debris. However, among these categories, only fragmentation debris is not fully monitored from ground-based observations. Through independent observation data, one can estimate the extent of its underrepresentation in the SSN catalog using an empirical calibration factor denoted as C_{TLE}, which is formulated as follows

$$c_{TLE} = 10^{\frac{1}{2}\exp\left(-2.464\left(\log_{10}\left(\frac{d_{SSN}}{[m]}\right)+1.22\right)^{2}\right)} \tag{15}$$

where the altitude-dependent detection diameter threshold d_{SSN} is fitted based on the observation data as follows (Klinkrad 2006).

$$d_{SSN}/[m] = \begin{cases} 0.089 & \text{for} \quad H \leq 620km \\ 10^{-2.737+0.604\log_{10}(H)} & \text{for} \quad 620\text{ km} < H \leq 1,300\text{ km} \\ 10^{-6.517+1.819\log_{10}(H)} & \text{for} \quad 1,300\text{ km} < H \leq 3,800\text{ km} \\ 1.0 & \text{for} \quad H > 3,800\text{ km} \end{cases} \tag{16}$$

To obtain a more accurate representation of the space debris environment in the LEO and lower MEO regions, corrections using Eq. (15) are necessary for ground-based SSN observation data. These corrections aim to reveal the accurate distribution of space debris at smaller catalog and sub-catalog sizes. The correction process demonstrates a normal distribution on a logarithmic scale of object diameters (d_{SSN}), with a peak calibration factor ($C_{TLE,max}$) of approximately 3.2 occurring at diameters around 6 cm, below the SSN's operational threshold. The correction diminishes to less than 5% for object diameters below 0.4 cm and above 100 cm. Consequently, due to this calibration factor (C_{TLE}), the modeled MASTER-2001 population with diameters greater than 10 cm comprises 17,832 objects, including 5,128 launch and mission-related objects, and 12,704 fragments of debris, compared to an unclassified catalog size of approximately 8,500 objects.

2.2.2 Fragmentations from Explosion and Collisions

Since the first documented on-orbit explosion of a Thor Agena D orbital stage in 1961, fragmentations have been the primary source of space debris within the size range from a few millimeters to a few decimeters. Objects within this range, particularly those with diameters between 1 cm and 10 cm, pose significant hazards as they cannot be shielded by current on-orbit technology and are not easily trackable by surveillance networks. The MASTER-2001 model estimates that there were approximately 370,000 fragments of such size in the year 2001, with around 142,000 of them located in LEO. Klinkrad (2006) provides a comprehensive overview of the chronological history, potential causes, and the most prevalent cataloged objects resulting from 175 on-orbit fragmentations up to January 2002. With the exception of three collisions, all fragmentations were due to explosions, of which only two were detectable in the GEO ring. The number of cataloged objects in GEO has typically ranged from 5% to 10% of the corresponding LEO population since 1970. Extrapolating from the assumption that fragmentation rates correlate with the number of satellites and upper stages, one could expect between 9 and 18 GEO fragmentations in total. Observations using the ESA space debris telescope (SDT) confirmed a substantial population of uncataloged objects in GEO, suggesting significant fragmentation events in that region. To enhance the 3D cell models for all recognized 3D evolutionary models, 11 simulated GEO explosion events were integrated alongside the two established GEO fragmentation events. The model utilizes a NASA breakup model (Johnson et al. 2001; Krisko et al. 2011), validated for debris sizes of 1 mm or larger, to simulate fragmentation events, cross-sections, masses, and imparted velocities. The modeling of the explosion and collisions can be found in (Patera 2008; Krisko et al. 2011; Lewis et al. 2011).

3. KOREAN SPACE DEBRIS MODEL

The Korean space debris model currently under development within KASI and KAIST includes both the 1D source-sink evolutionary model and the 3D spherical cell model. Fig. 4 shows the Korean space debris model's yearly development phase. The 1D source-sink model was first developed in 2023 and 3D onwards. The 1D source-sink model is primarily utilized to understand the effect of PMD together with the collision probability to propagate forward the debris count using NASA's standard break-up model. The 3D cell method is still under development, and is introduced in brief.

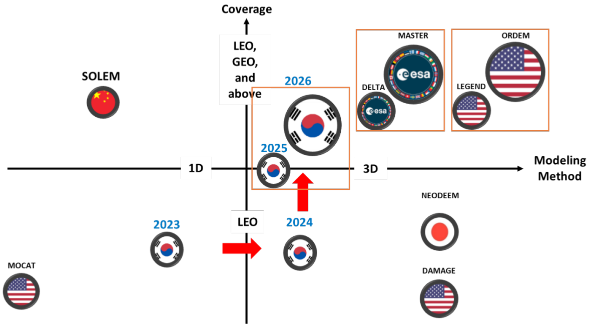

Fig. 4. Korean space debris engineering model development roadmap.

3.1 1D Space Debris Model for Post-Mission Disposal Analysis

The 1D space debris model takes the ordinary differential equation form introduced in Eq. (1). Using the source-sink model, we've performed space debris evolutionary analysis with varying PMD rates. Eq. (3) describes that the active satellite with PMD successfully performs the disposal at the end of the mission with a probability of PM. The disposal task fails with a probability of (1-PM), converting the active satellite to the derelict satellite. This failure increases the risk of collision between debris and derelict satellites.

The 1D simulation performed in this article shows the evolution of the space debris with varying PMD capable probability. We tested PMD success probability of 50%, 75%, and 99% with continuous increases in the active satellites with each respective PMD probability ratio. For the simulation, the launch was assumed to occur with the current trend of increase in launch frequency. Hence, the launch was assumed to occur every day by 2027 and within every 12 h in 2071 (Lubert 2018; Kulu 2021). Table 1 presents the parameters used for the simulation.

Figs. 5–7 show the space debris evolution with the parameters shown in Table 2 with PMD levels of 50%, 75%, and 99%. The initial active, derelict and debris population were taken from the Space-Track website (Space-Track nd).

Major differences in the population can be seen from debris and derelict objects in between different PMD success probabilities. The space population propagated with a PMD rate of 50% shows a slight increase in the derelict spacecraft count, but a significant increasing the debris near the altitude of 750 and above. Note that the newly launched

Table 1. Simulation parameters for 1D source-sink model

Variable	Value	Unit
$v_r(h)$	10	km/s
α	0.2	-
α_a	0.01	-
δ	10	-
H	$300 < H < 2{,}000$	km
δH	5	km
Active satellite life span	5	yr
λ	3,000	sat/yr

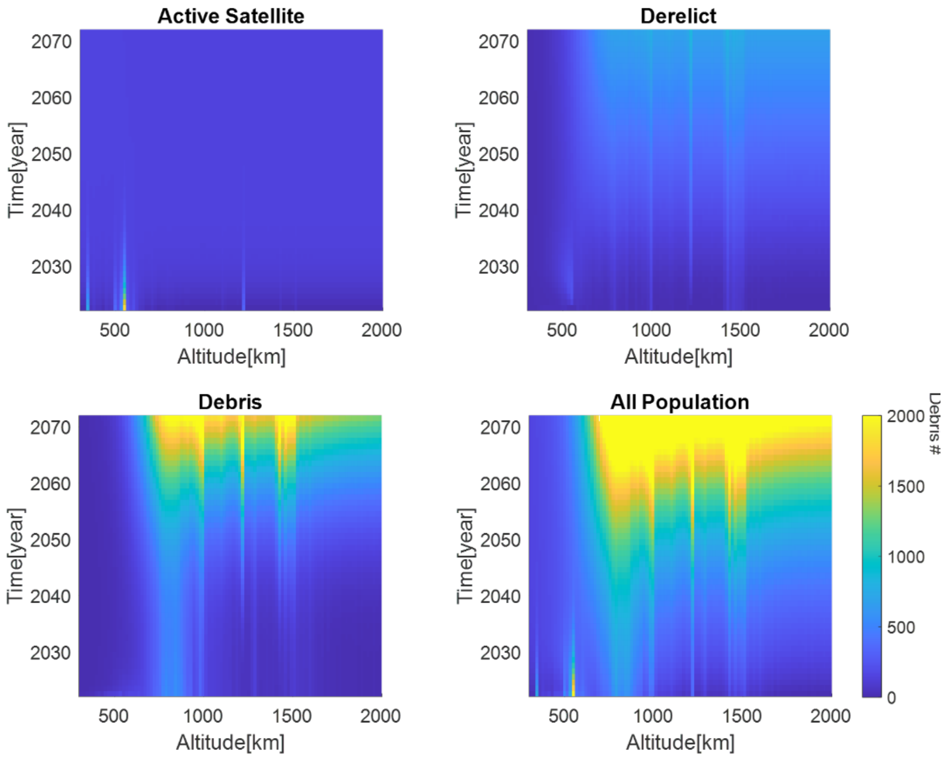

Fig. 5. 1D source-sink model with post-mission disposal (PMD) = 50%.

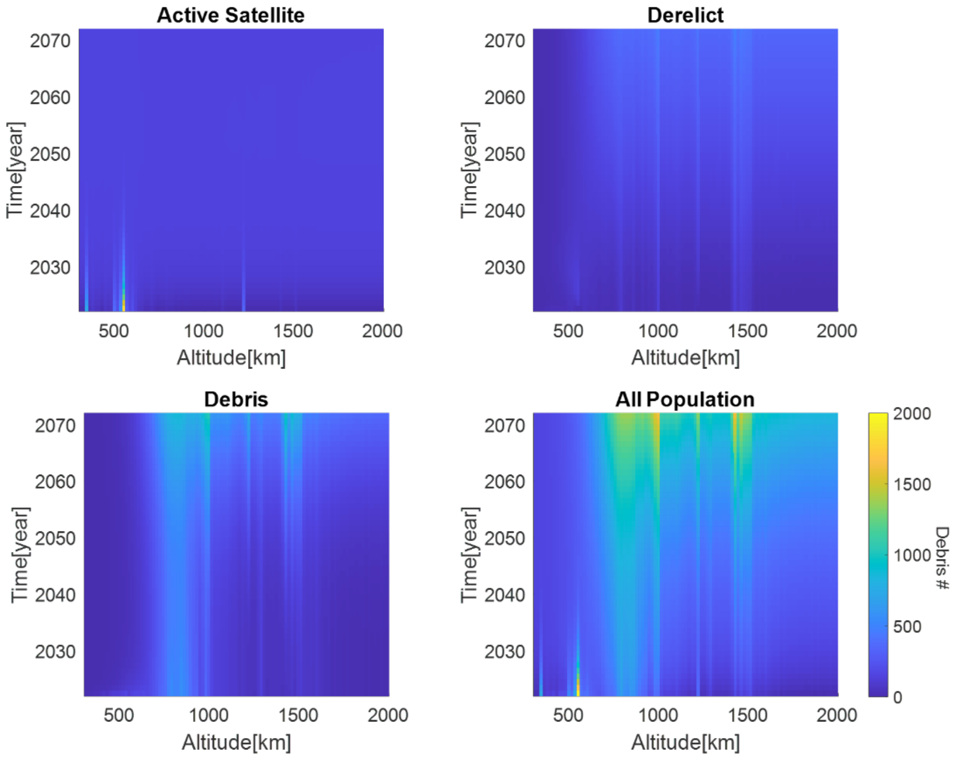

Fig. 6. 1D source-sink model with post-mission disposal (PMD) = 75%.

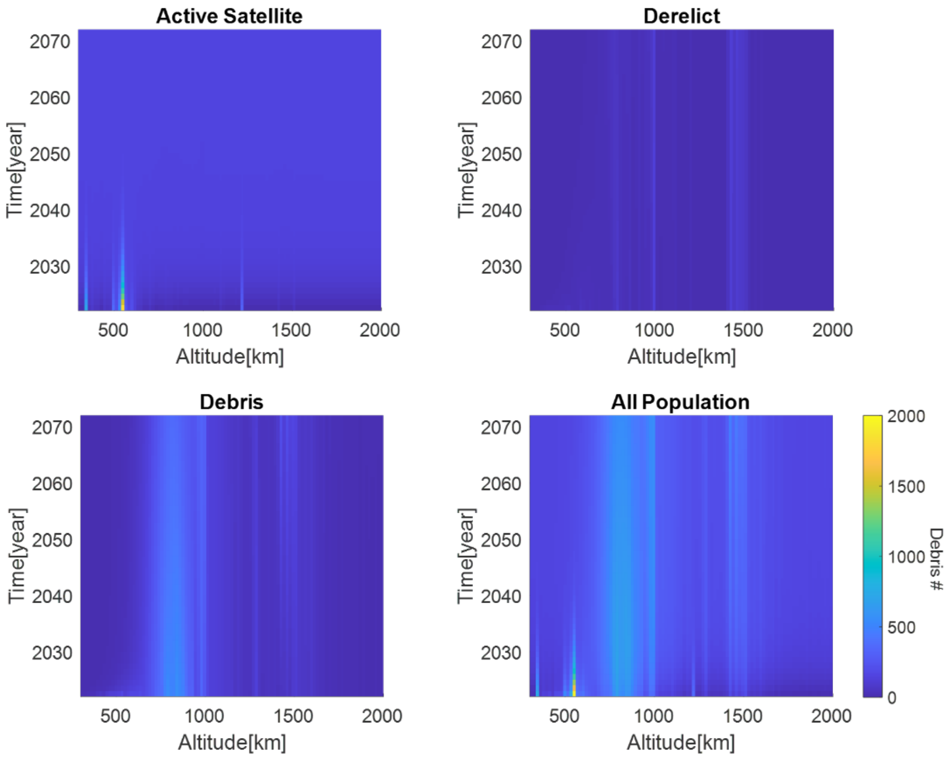

Fig. 7. 1D source-sink model with post-mission disposal (PMD) = 99%.

Table 2. List of currently developed space debris evolutionary and engineering models

	Space debris evolutionary model					Space debris engineering model		
	MOCAT	SOLEM	NEODEEM	DAMAGE	DELTA	LEGEND	MASTER	ORDEM
Debris modeling method	1D	1D	3D	3D	3D	3D	3D	3D
Country	USA (MIT)	China	Japan (JAXA, Kyushu University)	USA (University of Southampton)	ESA	NASA	ESA	NASA
Debris coverage	LEO	LEO	LEO-GEO	GEO	LEO-super-GEO	LEO-super-GEO	LEO-super-GEO	LEO-super-GEO

MOCAT, Massachusetts Institute of Technology (MIT) orbital capacity assessment tool; SOLEM, space object long-term evolution model; NEODEEM, near-earth orbital debris environment evolutionary model; DELTA, debris environment long-term analysis; LEGEND, NASA LEO-to-GEO environment debris 240; MASTER, meteoroid and space debris terrestrial environment reference; ORDEM, orbital debris engineering model; LEO, low Earth orbit; GEO, geostationary Earth orbit.

spacecraft with a 50% PMD failure rate is not sufficient enough to keep the space environment from the increase in space debris. With the increase in the PMD success probability, at 75%, the number of space debris compared to that of 50% success probability is significantly lower. At the PMD success probability of 99%, the space debris population shows a “vertical straight line,” noting that the spacecraft environment within the region is not growing but kept within the acceptable level.

3.2 3D Space Debris Model Development Status

The 3D debris model currently under development at KASI, in conjunction with the KAIST, utilizes the 3D space debris modeling method described in Chapter 2.1. The 3D debris model currently under development includes cataloged and non-cataloged space debris models. The cataloged debris model includes the trackable space debris listed on the Space-Track.org website. The TLE data is propagated utilizing the SGP4 (or SDP4 based on the population altitude) to retrieve the information related to the cells each population passes through within a given time.

The development model’s cell accuracy is divided into LEO, MEO, and GEO categories, where each follows the tabulates differences in the spherical coordinate (control volume) in Table 2, which follows the spherical cell division to that of MASTER developed by ESA. In each control volume, the TLE data from the space-track is propagated and determined by their passage conditions. Then, utilizing the Eq. (11)–(14), the flux information is stored in each cell. According to (Krisko 2014; Krisko et al. 2015), NASA's ORDEM stores flux information yearly, whereas the MASTER from ESA stores the flux information quarterly. We follow the MASTER's flux information frequency and update the flux information in a quarterly manner.

The breakup model developed within the 3D model follows the NASA's standard breakup model, discussed in Johnson et al. (2001) and Krisko (2011) and in Eq. (9). The velocity information of each cell is assumed to be a circular orbit within the identical cell radius range. The engineering model is expected to be developed such that the satellite's velocity is compared to that of each cell's common velocity direction to determine whether the collision is a catastrophic (f,c) or non-catastrophic (f,nc) collision event. The space debris population in the 3D debris modeling will include non-cataloged populations such as NaK debris, undetectable debris due to minute RCS from collisions and explosions, rocket motor slag and dust.

The orbital debris population models are described in Table 3. ORDEM 3.0 and MASTER-2009 both utilize theoretical and empirical basis methods with iterative approaches to model the space debris' probabilistic population growth. 3D Korean space debris model will include the published mathematical models of uncategorized space debris, such as NASA's break-up model. Korean space debris model's unique population data source is the OWL-Net (Table 4). The OWL-Net, an acronym for Optical Wide-field patroL Network, comprises 0.5-meter wide-field optical telescopes situated in various global locations such as Mongolia, Morocco, Israel, South Korea, and the USA. Each station operates uniformly under robotic control managed from its headquarters in Daejeon, Korea. Its primary aim is to gather orbital data for Korean satellites in LEO and GEO using optical methods and to ensure the stability of their orbital parameters. Additionally, the observation can be used to calculate more accurate flux information by correcting the orbital elements of the target spacecraft.

The development of the Korean space debris model faces several limitations that impact its precision and efficacy. One of the primary challenges is the lack of comprehensive radar and observation data. Effective space debris tracking relies heavily on accurate and consistent observational data, typically collected by ground-based radar and optical telescopes. South Korea's limited access to advanced tracking infrastructure constrains its ability to monitor the vast and dynamic environment of space debris accurately. The country's space debris modeling needs to be improved by more international collaboration and data sharing. The Korean space debris model relies on TLE information provided by Space-Track. However, space debris is a global issue that requires international cooperation and observation data sharing. Without robust international partnerships, South Korea's models may lack the comprehensive data to predict debris trajectories and potential collisions accurately. This lack of comprehensive data and collaboration makes it difficult for South Korea to develop precise and reliable space debris models, which are essential for safeguarding both national and international space assets.

Table 3. Control volume of 3D space debris model

	Altitude [km]	$\Delta\alpha$ [deg]	$\Delta\delta$ [deg]	Δr [km]	Total cell #
LEO	186–2,286	10	2	10	680,400
MEO	2,286–34,786	10	5	500	84,240
GEO	34,786–36,786	10	2	20	324,000

LEO, low Earth orbit; MEO, medium Earth orbit; GEO, geostationary Earth orbit.

Table 4. Space debris population of ORDEM 3.0, MASTER-2009, and 3D Korean space debris model

Parameter	ORDEM 3.0	MASTER-2009	3D Korean space debris model
Population storage	Yearly	Quarterly	Quarterly
Analysis interval	Yearly	Any interval	Any interval
Analysis time period	2010–2035	1957–2060	2024–2060
Orbital regime	LEO-to-GEO	LEO-to-GEO	LEO (2024), MEO-GEO (2026)
Population data sources	In-situ: STS returned surface Radar: SSN (TLE) Telescope: MODEST Ground tests: SOCIT4	In-situ: LDEF, EuReCa, HST returned solar panels Radar: SSN (TLE), TIRA, EISCAT Telescope: ESA-SDT Ground tests: SOCIT4, AEDC SRM burn SOURCE	Radar: SSN (TLE) Telescope: OWL-Net (Korean optical space surveillance system) (Jo et al. 2015; Choi et al. 2018; Park et al. 2018)

ORDEM and MASTER-2009 data from Krisko et al. (2015).
ORDEM, orbital debris engineering model; MASTER, meteoroid and space debris terrestrial environment reference; LEO, low Earth orbit, GEO, geostationary Earth orbit; MEO, medium Earth orbit; STS, space transportation system, SSN, space surveillance network; TLE, two-line element; LDEF, long duration exposure facility; HST, hubble space telescope; MODEST, Michigan orbital debris survey telescope; SOCIT, space object catalog identification task; TIRA, tracking and imaging radar; EISCAT, European incoherent scatter scientific association; ESA, European Space Agency; SDT, space debris telescope; AEDC, arnold engineering development complex; SRM, solid rocket motor.

4. CONCLUSION

This article has reviewed the most well-known space debris modeling method and is currently developing or already developed evolutionary and engineering models developed by space agencies and universities. 1D and 3D space debris modeling methods, specifically, the source-sink model and the 3D cell method, were described and compared their advantages and disadvantages. Based on the thorough analysis of the characteristics of 1D and 3D methods, The Korean space debris model was decided to include both the 1D and 3D space debris modeling methods to provide a comprehensive analysis. The 1D and 3D space debris evolutionary methods will be used to develop the 1D and 3D space debris engineering model, which will be the main contribution to the ongoing space debris research. Currently, the 3D modeling method is still under development, and the space debris engineering model is expected to be completed before 2027 in conjunction with the KASI and KAIST.

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