

Validation of Geostationary Earth Orbit Satellite Ephemeris Generated from Satellite Laser Ranging

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This study presents the generation and accuracy assessment of predicted orbital ephemeris based on satellite laser ranging (SLR) for geostationary Earth orbit (GEO) satellites. Two GEO satellites are considered: GEO-Korea Multi-Purpose Satellite (KOMPSAT)-2B (GK-2B) for simulational validation and Compass-G1 for real-world quality assessment. SLR-based orbit determination (OD) is proactively performed to generate orbital ephemeris. The length and the gap of the predicted orbital ephemeris were set by considering the consolidated prediction format (CPF). The resultant predicted ephemeris of GK-2B is directly compared with a pre-specified true orbit to show 17.461 m and 23.978 m, in 3D root-mean-square (RMS) position error and maximum position error for one day, respectively. The predicted ephemeris of Compass-G1 is overlapped with the Global Navigation Satellite System (GNSS) final orbit from the GeoForschungsZentrum (GFZ) analysis center (AC) to yield 36.760 m in 3D RMS position differences. It is also compared with the CPF orbit from the International Laser Ranging Service (ILRS) to present 109.888 m in 3D RMS position differences. These results imply that SLR-based orbital ephemeris can be an alternative candidate for improving the accuracy of commonly used radar-based orbital ephemeris for GEO satellites.

Keywords: geostationary Earth orbit (GEO) satellite, orbit determination (OD), orbital ephemeris, satellite laser ranging (SLR)

1. INTRODUCTION

While many geostationary Earth orbit (GEO) satellites have been launched for communications and environment monitoring, GEO satellites have become more engaged in satellite navigation systems (SNS); the Chinese BeiDou System (BDS), Indian Regional Navigation Satellite System (IRNSS), and Japanese Quasi-Zenith Satellite System (QZSS) have GEO satellites in their constellation to improve the quality of their navigation services. Precise orbital ephemerides of GEO satellites for SNS are generally obtained by a filtering process based on microwave navigation signals from ground monitoring stations. For a Compass-G1 of the BDS, its precise orbital ephemeris is generated to yield an accuracy on the order of several decimeters (Steigenberger et al. 2013). Zhou et al. (2011) estimated the orbits of BDS satellites to obtain meter-

level position errors with pseudo-range code measurements. Babu et al. (2015) performed orbital parameter estimation of IRNSS-1C GEO satellites to achieve meter-level residue between observed range measurements and computed range measurements. Mao et al. (2011) carried out orbit prediction of GEO and inclined-geosynchronous-orbit (IGSO) satellites based on orbit determination (OD) using combined pseudorange and phase data collected by 12 Chinese domestic ground stations to achieve several meter-level precision for one day. Shin et al. (2016) produced real-time OD simulations of GEO satellites based on inter-satellite ranging (ISR) for a preliminary study of a future Korean Navigation Satellite System (KNSS) to achieve OD accuracy within meter-level.

Unlike GEO satellites for navigation, precise orbital ephemerides of other GEO satellites are generally difficult to obtain. When there are no signal resources from the satellite to the

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ground station, observation data for OD is limited to passive observation systems such as radar or optical observations. Hwang et al. (2008) performed OD for a GEO satellite with a single ground station antenna and achieved 1.5 km root-sum-square in three-sigma position accuracy. Choi et al. (2015) conducted OD for a GEO satellite with angles-only optical tracking data to show that the position error was less than 2 km for a 2-day arc compared with two-line elements (TLE) from the North American Aerospace Defense Command (NORAD). TLE has been commonly used to obtain ephemerides of satellites since the orbital information inside TLE is consistently updated for all satellites (even for defunct satellites and space debris as well). Flohrer et al. (2008) analyzed the errors of TLE by categorizing the orbital characteristics of space objects. The orbital errors of TLE for GEO satellites in a one-day period were about 500 m in the along-track and cross-track directions and were less than 100 m in the radial direction. This may become aggravated when satellites are in a certain maneuver duration, which can result in a large error in the orbit propagation step. While the accuracy of TLE is known to be accurate enough for common GEO satellite missions, it may not be applicable to other missions that require more precise orbital ephemeris.

Satellite laser ranging (SLR) can be an efficient alternative approach for enhancing the predicted orbit accuracy of GEO satellites. While laser distance measurement systems have also been applied to satellite on-board relative navigation (Lee et al. 2015; Oh et al. 2016), SLR is organized with the ground station and a laser retro-reflector array (LRA) on the satellite bus to perform two-way ranging measurements. Since SLR is one of the best techniques for ranging between ground stations and satellites (Plotkin et al. 1965), it plays an important role for GEO satellites in validating the orbit accuracy determined from other observation methods. Zhao et al. (2013) conducted precise OD of BDS satellites using SLR to obtain meter-level position accuracy. Oh et al. (2017) performed OD of elliptical-inclined-geosynchronous-orbit (EIGSO) and GEO satellites and demonstrated decimeter-level position errors. Deng et al. (2014) processed SLR data to assess the quality of BDS satellites. The aforementioned research has demonstrated the advantages of SLR observations for verifying the orbits of GEO satellites. However, there has been no previous research about SLR-based orbit prediction for GEO satellites. If the SLR-based predicted orbital ephemeris would become more precise than the conventional TLE-based ephemeris, the SLR technique may not only be used to as an auxiliary method verify estimated orbits from other observations, but also could work as a main OD strategy.

In the near future, Korea plans to payload an LRA onto a GEO-Korea Multi-Purpose Satellite (KOMPSAT)-2B (GK-2B) to obtain SLR observation data from the Geochang

domestic tracking site. While previous Korean GEO satellites maintained their orbital accuracy within 18 km, the GK-2B mission requirement for OD is about 3 km with two domestic and international tracking sites (Park et al. 2018). A higher accuracy of orbital ephemeris would widen the applicability of GEO missions. To determine the feasibility of SLR-based precise orbital ephemeris of GK-2B, this study generated orbital ephemeris using an SLR-based OD solution and analyzed its accuracy.

This paper is structured as follows: Section 2 describes specific parameters and information about the target GEO satellites selected in this study, and presents SLR-based OD results as well. Section 3 presents the ephemeris generated from SLR-based OD and its validation. Real-world verification with Compass-G1 is also discussed. Section 4 draws conclusions.

2. SATELLITE ORBIT DETERMINATION

2.1 Overview of GK-2B

GK-2B is the third Korean GEO satellite for ocean and environmental monitoring, which succeeded the previous Communication, Ocean, and Meteorological Satellite (COMS) mission managed by the Korea Aerospace Research Institute (KARI). It is equipped with an advanced meteorological imager (AMI) and the Korean space environment monitor (KSEM) as primary payloads, and is scheduled to be launched in the year 2019 with 10 years of mission lifetime. It is to be at 128.408 degrees East in longitude and 36,000 km in altitude (Park et al. 2018). Unlike GK-2A, the first GK-2 mission which has a continuous 24-hour mission schedule, GK-2B is scheduled to operate during daytime only. Therefore, to avoid any possible risk of damaging the optical instruments mounted on GK-2B, precise ranging by SLR between ground tracking sites and the satellite is to be performed at night with only domestic SLR stations, and with the shutters of the payloads closed (Park et al. 2018). LRA will be mounted on the satellite bus to obtain SLR data. The primary objective of SLR is to maintain the precise OD while validating the SLR technology for future domestic satellite missions. Specific satellite parameter information for GK-2B are found in Oh et al. (2018). Fig. 1 shows the modeling structure of GK-2B.

International SLR tracking sites will be prohibited from ranging GK-2B to protect the sensitive optical instruments on the satellite; only the domestic Geochang SLR station at Mt. Gamak, which was developed by the Korea Astronomy and Space Science Institute (KASI), will observe GK-2B. Fig. 2 shows the Geochang SLR station. With a divergence angle of 8 arcsec and a nominal laser repetition rate of 60 Hz for SLR,

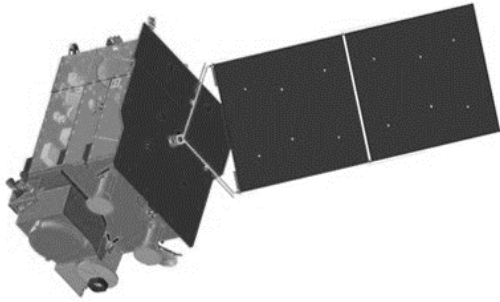


Fig. 1. GK-2B GEO Satellite (Image Courtesy of KARI).



Fig. 2. Geochang SLR station at Mt. Gamak.

the station is currently in the initialization/calibration stage and will be in normal operation from the end of the year 2018; normal operation is expected when GK-2B is in orbit (Lim et al. 2017).

2.2 SLR-based Orbit Determination

For generating precise orbital ephemerides, SLR data simulation and the OD process were proactively performed by utilizing Orbit Determination ToolKit (ODTK) version 6 of Analytical Graphics Inc. (AGI). Geochang station was expected to be in normal operation and hold similar measurement errors to those of conventional SLR stations. Gaussian white noise with 4 mm one-sigma error was applied to the SLR normal point (NP) data generation, considering that the amount of white noise sigma is usually between about 1 mm and a few cm for SLR measurements (Vallado et al. 2014). The OD process was performed by iterating the forward filtering process and the backward smoother (Wright & Woodburn 2008; Vallado et al. 2010). Specific satellite parameter information for GK-2B and the OD settings can be found in Oh et al. (2018). Two scenarios were constructed to analyze the effect of the number of SLR measurements on the OD accuracy. For a one-week arc,

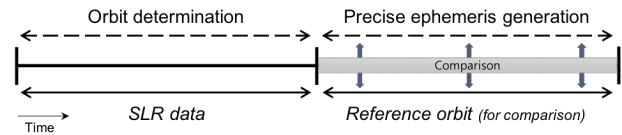


Fig. 3. Orbital ephemeris generation and comparison process.

the sparse case collects 9 NPs every day from the Geochang SLR station, while the dense case collects 13 NPs. In this paper, we consider the dense case with 91 NPs in one week. The OD errors of GK-2B were evaluated by directly differencing the determined orbit and the simulated (true) reference orbit for one week-length period to yield 43.46 m and 24.01 m as 3D root-mean-square (RMS) values for the sparse and the dense case, respectively.

3. ORBITAL EPHEMERIS GENERATION AND VALIDATION

3.1 GK-2B Ephemeris Validation

Based on the OD results presented in Section 2, the predicted ephemeris was generated and compared with the reference orbital ephemeris of GK-2B. Fig. 3 shows the concept of the ephemeris generation and comparison process. OD is performed during the former period in which SLR data exist. Then, the precise ephemeris is generated in the latter period and compared with the reference orbit.

The reference orbit is proactively generated by the true simulated orbit of GK-2B and compared with the predicted ephemeris. The predicted ephemeris is generated by considering the consolidated prediction format (CPF) of the International Laser Ranging Service (ILRS). It is commonly used for formatting SLR-based precise orbital ephemeris and provides satellite orbital state information in tables of X, Y, and Z positions. Users can interpolate the CPF data to precisely locate the satellite at a certain time. Generally, it contains a seven-day length orbit, which could otherwise cause problems in the accuracy of the orbit over time (Ricklefs 2006). The reason why the CPF has such a long prediction time is to prepare for situations in which daily updates are not available for some satellites. Since the general CPF orbit update rate is one day, the minimum effective length of a precise orbit is the same as the update rate. Table 1 presents the dynamic model parameters used for ephemeris generation. The International Celestial Reference Frame (ICRF) is used as the reference coordinates; Runge-Kutta-Fehlberg 7(8) (RKF 78) is used for integration; drag is not included in the dynamic model for GEO satellites; a spherical model is adopted for solar radiation pressure (SRP) with $C_r A/M$ additive correction, where

Table 1. Dynamic Model Parameters for ephemeris generation of GEO satellite

Parameter	Value
Ephemeris Length	1 day
Step size	1 minute
Coordinate frame of predicted ephemeris	ICRF
Gravity model	GGM03C 30×30 (Tapley et al. 2007)
Nutation method	IERS 1996 (McCarthy 1996)
Numerical Integration	RKF 78
Drag	Not applied (for GEO satellite)
Solar radiation pressure	Spherical model with $\frac{C_r A}{M}$ additive correction
Third body gravity	Sun, Moon, all planets and Pluto with JPL DE 405 (Standish 1998)

Table 2. Precise orbital ephemeris errors of GK-2B (1 day)

	Radial Difference	In-Track Difference	Cross-Track Difference	3D Difference
RMS	3.079 m	17.056 m	2.121 m	17.461 m
Maximum	4.015 m	23.800 m	3.005 m	23.978 m

Table 3. Precise orbital ephemeris errors of GK-2B (1 week)

	Radial Difference	In-Track Difference	Cross-Track Difference	3D Difference
RMS	4.384 m	24.919 m	2.123 m	25.391 m
Maximum	7.491 m	44.661 m	3.007 m	44.759 m

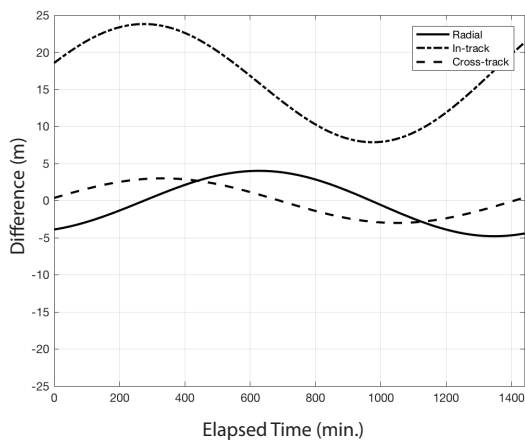


Fig. 4. Precise orbital ephemeris errors of GK-2B (1 day).

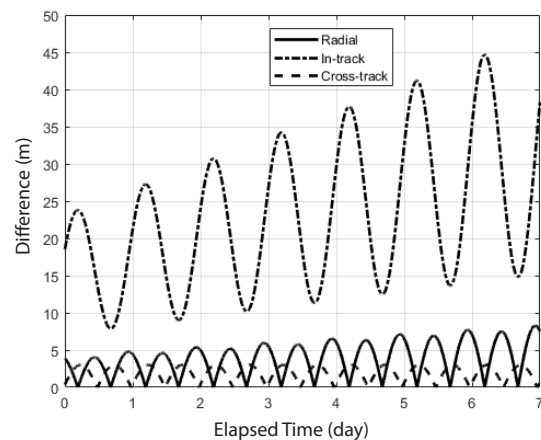


Fig. 5. Precise orbital ephemeris errors of GK-2B (1 week).

C_r is the SRP coefficient, M is the satellite mass, A is the effective area of the satellite; the gravity of the sun, moon, all planets, and the dwarf planet Pluto are all incorporated.

Fig. 4 and Table 2 show the comparative results between the reference orbit and the predicted orbital ephemeris. The starting epoch of the predicted ephemeris is 09:00:00 6 July 2020 UTC, which coincides with the end of the OD period. The errors of one-day prediction in terms of RMS in the radial, in-track, and cross-track directions are 3.079 m, 17.056 m, and 2.121 m, respectively. The overall 3D RMS error is 17.461 m. According to the simulation results, the accuracy of SLR-based predicted ephemeris for GK-2B remains precise for a day. OD errors were found to be less than those of the predicted ephemeris; while the OD errors were calculated for the period of one week, prediction is performed only for one day followed by the OD process. This implies that the

SLR-based orbit prediction maintains its accuracy for a one day period. When the period of prediction was lengthened to one week, the accuracy of the predicted ephemeris showed gradual degradation, as shown in Fig. 5 and Table 3. The RMS errors in the radial, in-track, and cross-track directions, and 3D grew to 4.384 m, 24.919 m, 2.123 m, and 25.391 m, respectively.

3.2 Compass-G1 Ephemeris Validation

In an attempt to justify the simulational validation of GK-2B ephemeris, verification of the OD and ephemeris generation strategy in “real world” conditions was carried out. The reference OD results for the ephemeris generation are derived from previously published research (Oh et al. 2018). The Chinese Compass-G1 GEO satellite was selected,

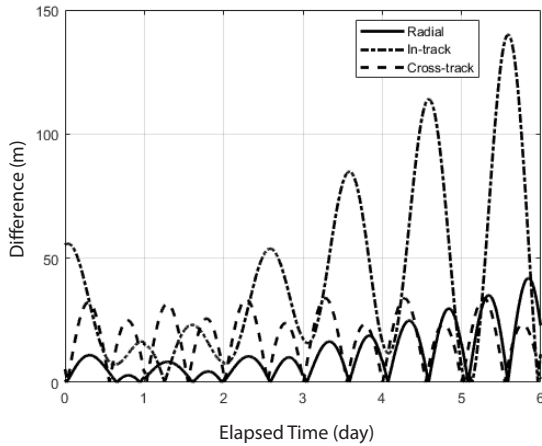


Fig. 6. Differences between SLR-based ephemeris and GFZ final orbit of Compass-G1 (1 week).

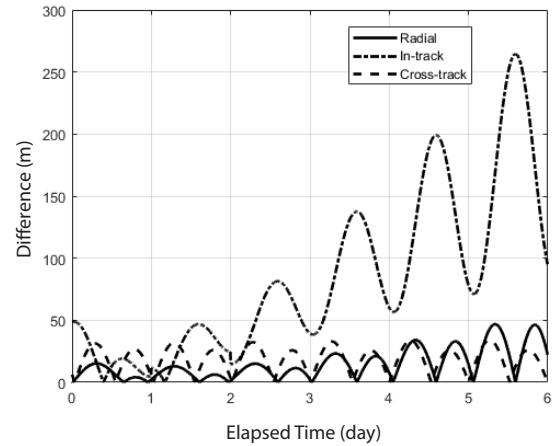


Fig. 7. Differences between SLR-based ephemeris and ILRS CPF of Compass-G1.

Table 4. Differences between SLR-based ephemeris and GFZ Final orbit of Compass-G1 (1 week)

	Radial Difference	In-Track Difference	Cross-Track Difference	3D Difference
RMS	15.166 m	54.242 m	20.617 m	59.977 m
Maximum	41.996 m	140.125 m	33.926 m	140.348 m

Table 5. Differences between SLR-based Ephemeris and ILRS CPF of Compass-G1

	Radial Difference	In-Track Difference	Cross-Track Difference	3D Difference
RMS	18.859 m	106.221 m	20.900 m	109.888 m
Maximum	46.387 m	264.554 m	26.688 m	264.850 m

as its precise ephemeris was provided continuously without disruption. The OD of Compass-G1 was performed with the same strategy as that used for GK-2B. The SLR data from the Changchun Chinese SLR station was applied from October 21 to 24, 2017. The 3D errors of OD of Compass-G1 were 31.89 m in RMS.

Based on the above ODs from the previous research, the predicted orbital ephemeris of Compass-G1 was generated. It was then externally compared with two different orbital sources: the final orbit from the International Global Navigation Satellite System (GNSS) service (IGS) and the ILRS CPF orbit. GeoForschungsZentrum (GFZ), one of the IGS analysis centers (AC), was selected as the final orbit provider. The accuracy of the GFZ final orbit was qualified by direct comparison with the ephemeris made by Wuhan AC to yield errors of about 3 m (Deng et al. 2014). Also, the orbit overlap errors of Compass-G1 were 27.9 cm, 86.7 cm, and 84.5 cm in the radial, along-track, and cross-track directions, respectively (Uhlemann et al. 2013). The period of comparison was set to six days from October 25 to 30, 2017, as no GFZ final orbit is available for October 31, the seventh day.

Fig. 6 and Table 4 present the differences between the generated precise orbital ephemeris for Compass-G1 and the GFZ final orbit. The RMS errors are 15.166 m, 54.242 m, and 20.617 m for the radial, in-track, and cross-track directions, respectively. The maximum 3D error is 140.348 m

over the whole prediction period. These differences are more significant than the error of the GK-2B simulation, as it should be due to the unmodeled dynamic models and measurement errors that exist in the real world. Despite the increasing differences as time elapses, these results indicate that the predicted orbital ephemeris based on the OD with SLR measurements maintained decimeter level accuracy for a one-day period.

Fig. 7 and Table 5 present another external comparison of the Compass-G1 ephemeris with ILRS CPF. Since ILRS CPF provides 7-day orbits every day, the first day of CPF was selected. The overall 3D differences in RMS are 109.888 m. This is still quite large compared to the previous comparison with the GFZ orbit. However, as the orbit accuracy of the ILRS CPF is not guaranteed to be precise, this result may not indicate the accuracy of the SLR-based predicted orbital ephemeris.

4. CONCLUSIONS

To determine the feasibility of SLR measurements for generating orbital ephemeris of GEO satellites, predicted orbital ephemeris for GK-2B and Compass-G1 were generated. From the GK-2B simulation, the 3D errors of generated ephemeris were less than 23.978 m for one-day prediction.

The ephemeris generation strategy was verified through real-world verification with a Compass-G1 GEO satellite. The generated predicted orbital ephemeris of the Compass-G1 was externally compared with the final orbit based on GNSS signals and the CPF from the ILRS. While the results showed differences of hundreds of meters compared with the CPF orbit for a week, 3D differences between the results and the final orbit from the GFZ AC were less than 56.001 m for one day. Since the accuracy of the final orbit is known to be below the decimeter level, the accuracy of SLR-based predicted orbital ephemeris is applicable to the maintenance of GEO satellite orbits. Also, SLR-based ephemeris can meet the 3 km orbit accuracy requirement of GK-2B. Furthermore, this result will help to expand the applicability of future GEO satellites, in that it can maintain precise orbits of GEO satellites without any direct data transmission/reception between satellites and ground stations.

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